Exploring the Natural Order of Operations with Bingo Chip Arrays

Part 3 of 6 from the Unit: Deepening Understanding of Order of Operations

A Common Core-Aligned Lesson Plan to use in your Classroom

Author
Connie Rivera, Capital City YouthBuild

The activity

Students will naturally use multiplication before addition and subtraction and write expressions or equations using arrays. Reiterating an acronym (such as PEMDAS) to remember part of a procedure is not nearly as useful in the long run as teaching for conceptual understanding. Students often remember the memory aids, but not all of the aspects of them (such as multiplication and division are taken together from left to right), when to use them, or why they work – therefore limiting their understanding for higher instructional levels.

This activity will help them build a foundation for future instruction. An activity such as this gets away from worksheets and makes students think beyond procedure.

Students will

- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. CCS 2.OA.4
- Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. CCS 3.OA.1
- Apply properties of operations as strategies to multiply and divide. (commutative property of multiplication,... associative property of multiplication,... distributive property) CCS 3.OA.5
- Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. CCS 5.OA.1
- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. CCS 5.OA.2
- Write and evaluate numerical expressions involving whole-number exponents. CCS 6.EE.1
- Shift 3 – Rigor (conceptual understanding)
Order of Operations Lesson 3: Exploring the Natural Order of Operations with Bingo Chip Arrays

Resources needed:

- PowerPoint Presentation "Exploring the Natural Order of Operations with Bingo Chip Arrays"
- Photograph “Number of Candies?” included at the end of this lesson plan
- 1 bag of 25 bingo chips (could use drawings instead of chips for a model)
- 1 bag of bingo chips, pennies, or crackers per student. Make 3 bags of each prime number beginning at 17 [continue with 19, 23, 29, 31, 37, 41, 43, etc.] until you have enough for each participant to have one bag. Print directions and tuck into each bag.
- 2 – 3 sheets blank or lined paper per participant

Approximate time:

45–60 minutes

Instructions

Academic Vocabulary: What domain or academic-specific words will you use in the lesson?

- array – a set of objects arranged in a specific pattern (usually rows and columns)

1) Opening Discussion:

a) Show the Number of Candies photo. “How many candies are in this picture?” 12. “How did you figure that out?” If necessary, add: “Is there a way to know without counting?” Write their expression or equation up on the board (such as $3 \times 4 = 12$). “Could I have written it differently?” (such as $4 \times 3 = 12$)

- “The arrangement of the candy in the picture almost looks like a square, but it’s actually arranged in a rectangle. These wouldn’t be the most obvious expressions for this array, but can I use an exponent to describe the array?” ($3^2 + 3$ or $3 + 3^2$ or $4^2 - 4$)
• “How can I find out how many chips are in this bag without actually counting every chip?” Using the 25 bingo chips from the bag, make an array with 2 rows of 10 and a row of 5.

```
   □□□□□□□□□□
   □□□□□□□□□□
   □□□□□□□□□□
```

“How can I write an expression for that?” Solicit ideas to get at least three versions.

For example: $2 \cdot 10 + 5$

```
2 + [□□□□□□□□□□
    □□□□□□□□□□]
10
```

$3 \cdot 10 - 5$

```
10
  □□□□□□□□□□
  □□□□□□□□□□
3
```

$3 \cdot 5 + 2 \cdot 5$

```
3 + [□□□□□□□□□□
   □□□□□□□□□□
   □□□□□□□□□□]
   □□□□□□□□□□
5
```

• Practice with a new arrangement (such as a 4 by 6 array with one on an end) if needed.

• Demonstrate square arrays that lend themselves to using exponents. Begin with a 5 by 5 arrangement to solicit an expression of $5^2$.

```
  □□□□□
  □□□□□
  □□□□□
  □□□□□
  □□□□□
```

Practice a few other arrangements that allow students to experience seeing “squared as an actual square shape, and connect that to an exponent of 2.

$5^2 + 4 \cdot 2 + 1$
2) **Activity 1 (individual):**

   a) Pass out unmarked bags of bingo chips randomly. Tuck the following directions into each bag:

   **Writing Expressions**
   1. Arrange these items so that you can find out how many there are in your bag **without** actually counting every object.
   2. Draw a representation of your array on paper.
   3. Write a numerical expression to represent your array.

3) **Activity 2 (small group):**

   a) Form groups by asking participants to find the others who have the same number of bingo chips as they do.

   b) Once in groups, participants will compare their arrays and numerical expressions. Working together, they will determine if there are any other possible arrangements for their number of bingo chips. Next, ask participants to draw the arrangements and write equations to go with them. Pass out the following written directions as needed:

   **Writing Equations**
   In your group:
   1. Compare your arrays and numerical expressions.
   2. Determine if there are any other possible arrangements. If so, draw the arrangements and write equations to go with them.

3) **Extension:** Using the bingo chips from Activity 1, represent given expressions with an array. Do at least one from each row.

   
   
   \[
   \begin{align*}
   2 + 3 \cdot 5 & \quad 5 + 2 \cdot 3 & \quad 4 + 6 \cdot 3 & \quad 3 + 5 \cdot 3 \\
   2 \cdot 4 + 1 & \quad 5 \cdot 2 + 3 & \quad 4 \cdot 3 + 6 & \quad 3 \cdot 5 + 2 \\
   3 \cdot 2 + 5 \cdot 2 & \quad 3 \cdot 3 + 4 \cdot 2 & \quad 4 \cdot 3 + 2 \cdot 2 & \quad 2 \cdot 4 + 2 \cdot 3 \\
   3 \cdot 5 - 4 & \quad 4 \cdot 3 - 2 & \quad 2 \cdot 4 - 1 & \quad 9 \cdot 1 - 4 \\
   4^2 + 1 & \quad 2 + 3^2 & \quad 3^2 + 6 & \quad 4^2 - 2
   \end{align*}
   \]
4) **Evidence of Success (Formative Assessments):** What exactly are students expected to be able to do by the end of the lesson, and how will mastery be measured? What would outsiders see that would convince them that students have developed a deep understanding?

   a) Success will be observed as students are able to write multiple numerical expressions that represent the same arrays and create arrays to represent given expressions. (Formative Assessments): What exactly are students expected to be able to do by the end of the lesson, and how will mastery be measured? What would outsiders see that would convince them that students have developed a deep understanding?

   b) Success will be observed as students are able to write multiple numerical expressions that represent the same arrays and create arrays to represent given expressions.

**Differentiated Instruction**

It would be helpful to visual learners if students were able to see the arrays created during the opening discussion on a document camera, drawn on the board, or even by circling around a table. See Success Tips for additional ideas.

**Success Tips**

- When demonstrating examples, eliminate any unnecessary parenthesis.

- Be sure to use context when reading expressions. For example, 3 • 4 can be read: “Three rows of four.” “Times” does not have as much meaning.

- Avoid mentioning “the Order of Operations” or “PEMDAS (Please Excuse My Dear Aunt Sally)” so that students who do not make the connection will not rely on past habits that have not been beneficial to full understanding.

- When circulating during the activities:
  - Prompt different expressions by asking if students can use the same array they wrote a multiplication/addition expression for to demonstrate multiplication and subtraction. In the same way, prompt use of exponents.
  - Foster deeper connections by asking students why they multiplied first to find out their total.
  - Deepen students’ thinking by rotating their array 90° and asking if that changed the expression they would write for it.
Number of candies?