Building Procedural Skill and Fluency with the Order of Operations

Part 6 of 6 from the Unit: Deepening Understanding of Order of Operations

A Common Core-Aligned Lesson Plan to use in your Classroom

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The activity

Students will apply their understanding of the order of operations to problems that require them to:

- build visual arrays, creating a concrete model to represent equations
- find visual arrays equivalent to expressions and expressions equivalent to other expressions
- reason about the effect of different values and operations on an expression
- identify patterns and apply properties of numbers to problem solving
- creatively problem-solve, applying the properties of numbers and the order of operations

Exploring the reason behind this accepted principle (the order of operations) helps us unlock the creativity of math, giving students the flexibility to solve problems in a variety of ways. By exploring a selection of examples that get at different aspects of the order of operations (properties of numbers, grouping symbols, visual representations), students have a chance to demonstrate what they’ve learned. This solidifies the foundation of their conceptual understanding.

Students will

- Write and evaluate numerical expressions involving whole-number exponents. CCSS 6.EE.1
- Apply the properties of operations to generate equivalent expressions. CCSS 6.EE.3
• Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). CCSS 6.EE.4

• Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure... They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. CCSS MP.7

• Make sense of problems and persevere in solving them. Mathematically proficient students can explain correspondences between equations, verbal descriptions, ... or draw diagrams ... using concrete objects or pictures to help conceptualize and solve a problem. CCSS MP.1

• Students with a solid conceptual understanding see mathematics as more than just a set of procedures. They know more than “how to get the answer” and can employ concepts from several perspectives. Shift 3 – Rigor

Resources needed:
• Photocopies:
  • Warm-Up:
    ▪ Graduation Celebration (optional hand-out during warm-up)
  • Stations:
    ▪ Expressions and Array – Card Match
    ▪ Order of Operations – Dice Game
    ▪ Properties of Numbers Challenge
    ▪ Thirty-Nine
    ▪ Prove It! True or False
  • Individual Practice Sheets:
    ▪ Find the Missing Number
    ▪ Grouping Symbols
    ▪ Creating True Equations
• Video The Order of Operations is Wrong... Morally Wrong that is from 4-minute Physics which can be played through YouTube

  • 4 - 6 dice (available at Dollar Tree stores – 10 for $1)
  • 100 pennies
  • 1 calculator per student
  • 1 pencil per student
  • 2 – 3 sheets of lined paper per student
  • 1 – 2 sheets of blank paper per student
  • 5 – 10 markers in 3 – 4 different colors
  • timer or stop-watch
  • small incentive prizes, optional (Kisses, miniature decks of cards, pencils, erasers, mini-bubbles, etc.)
  • consider purchase of 24Game for follow-up (can be done with a deck of cards)

Approximate time:
1 hour
Order of Operations Lesson 3: Exploring the Natural Order of Operations with Bingo Chip Arrays

Instructions

Academic Vocabulary: What domain or academic-specific words will you use in the lesson?

- *expression* – a math phrase made of numbers, symbols, and operation signs grouped to show the value of something
- *equation* – a written statement showing equality between two expressions

1) Warm-Up:

a) On the board or screen before class, put up the following number sense problem:

Graduation Celebration!

- YouthBuild has decided to spend a fixed amount celebrating your successful completion of the program. You may choose from the following dollar values equal to:
  - double your age, squared, or
  - 2 to the exponent of your age, or
  - the square root of your age, cubed.

- *Without actually calculating* these, which option would you choose? Thoroughly explain your choice.

b) Ask students to write a few sentences on lined paper (or the optional handout provided) explaining their choice.

c) “Now, use a calculator to decide if you are pleased with your choice.”

d) “Did anything surprise you?” “Would you like to change your answer?”

e) Allow time for students to explain a new answer. Now, break the news to them that YouthBuild will likely *not* be spending that much money on a celebration (or even their teacher’s salary)! Ask your youngest student and your oldest student how much money they had for their answer. “Why is [oldest student]’s answer so much larger than [youngest student]’s answer?”
f) If the class needs more time with variables, bring out a pile of bingo chips for each student and practice by giving students an expression to represent. Because there is a variable, different interpretations are possible. For example, you could call out $4r + 3$, where the $r$ stands for the number of rows.

Here $r$ is 5.  

$\dots$and here, $r$ is 8.

- While the bingo chips are out, you can consider reviewing squared numbers.

2) Opening Discussion:

a) “Today you will be visiting stations with a partner or in small groups. Afterwards you will get a set of Individual Practice Sheets that will demonstrate how you’ve Deepened Your Understanding of the Order of Operations. (adjust if you decided to give these out ahead of time) There are many pieces to that – the properties of operations, operations signs, grouping symbols, visual representations of what expressions mean… Before we begin, we are going to run through a few examples that will be similar to tasks you will see at the stations and on your individual practice sheets.”

<table>
<thead>
<tr>
<th>Using parenthesis, write this problem 4 different ways.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write it so that it equals 1, 4, 14, and 18.</td>
</tr>
<tr>
<td>$20 \div 5 + 5 \times 2 = $</td>
</tr>
</tbody>
</table>

- There are many ways for students to arrange parenthesis to get an answer of 14, and fewer ways to get the other answers. Review student answers. You may consider following up by asking students to insert parenthesis to make this statement true:

$$3 + 2 \times 15 - 7 = 19.$$
b) Continue with this problem:

\[
3 \times 4 + 5 \text{ equals } 4 \times 5 - 3
\]

Draw a visual representation to show that this is true.

c) Read the problem with context (three sets of four) rather than the word “times.” An example of how \(3 \times 4 + 5\) could be drawn follows:

An array of 3 columns of 4 plus 5 compared to an array of 4 columns of 5 with 3 removed:

```
****
****
****
****
```

```
****
****
****
****
```

has the same shape, only mirrored.

d) You might ask students to do both representations. Next, (optional) challenge them to prove that this will work for any 3 consecutive positive whole numbers. One proof:

\[
n(n+1)+n+2 = (n+1)(n+2) - n
\]

\[
n^2 + n + n + 2 = n^2 + 2n + n + 2 - n
\]

\[
n^2 + 2n + 2 = n^2 + 2n + 2
\]

3) **Lesson/Stations:**

a) Set out the pre-prepared stations around the classroom. Give students an overview of each station as you set it out. Let students know that they are expected to be working the whole time, but that they are not necessarily going to have enough time to finish each station in the time they are given.

b) Pair or group the students. Allow 10 - 12 minutes for each station (set an alarm or timer).

c) Optional incentives – a Kiss for completion of each station, a prize for the members of groups with the most answers to *Thirty-Nine*, a prize for accurately completing the most stations.
Expressions and Array – Card Match

- **Description:** Students must match 2 expressions and one visual array to make 5 sets of cards.

- **Heads Up:** Students may be tempted to match cards by solving expressions and counting arrays. Ask them to find a different way to match the cards, using this method as a check only if absolutely necessary.

- **Materials:** 1 – 2 sets of cards printed on card stock and cut apart, scrap paper, and pencils, directions for students (saved from top of hand-out)

Order of Operations – Dice Game

- **Description:** Students will fill the numbers into an expression with fixed operators, reasoning about the effect of different values for the largest possible number.

- **Materials:** 1 photocopy of both game cards for each student, 2 – 4 dice, pencils

Properties of Numbers Challenge

- **Description:** Students will identify patterns in empty equations and apply properties of numbers to determining the numbers.

- **Materials:** 1 photocopy of the Properties of Numbers Challenge for each student, pencils

Thirty-Nine

- **Description:** Students will use fixed numbers, any operation or grouping symbol, and their understanding of the properties of numbers and the order of operations to create equations that equal 39.

- **Heads Up:** The examples at the bottom of the page are provided only as extra practice on the off chance that groups may finish early or to take as homework.

- **Materials:** 1 photocopy of the first page of Thirty-Nine hand-out per student, 2 – 3 sets of the cards on card stock and cut out (optional), pencils
Prove It! True or False

- **Description**: Students will build a visual array to prove or disprove each equation.
- **Materials**: 1 photocopy of *Prove It! True or False* for each student, 100 pennies, blank paper, 5 – 10 markers in 3 – 4 different colors.

4) **Wrap Up**:

a) In order to see how much each student has learned, ask students to complete the following Individual Practice Sheets:

- *Find the Missing Number*
- *Grouping Symbols*
- *Creating True Equations*

b) Any tasks (from stations or individual practice) that are not completed can be sent home if it is at a student’s independent level, or saved for completing in class another day.

c) Finish class by showing the video: *The Order of Operations is Wrong... Morally Wrong that is*, from Minute Physics which can be played through YouTube (4:11 minutes). Congratulate students on meeting our goal of deep understanding of the concepts rather than blindly following procedures that they previously couldn’t explain. Remind them that they have built a foundation for understanding algebra.

5) **Evidence of Success (Formative Assessments)**:

- This lesson is full of practice sheets and activities that can be viewed to determine a student’s level of understanding.

**Differentiated Instruction**

You may consider dividing the stations and completing them on separate days for learners who may get overwhelmed or who are not likely to do homework. You can extend the time given for stations and visit less of them. Students can be given the Individual Practice Sheets as homework before this class period.

Suggestions for challenging questions and extra practice are included in the Opening Discussion section of this Lesson.
**Success Tips**

- Answers were only provided to half of the problems in this lesson so that you can try the problems as part of preparing for the lesson.
- At this point it’s okay to use the phrase “the Order of Operations.”
- Continuing to include these ideas in your teaching so that it does not become an isolated skill is important. Consider using the 24Game as a warm-up several days a week following this unit. See the *24* by Roeann Nelson, Teacher Fellow 2012 – 2013, from YouthBuild Columbus.